Math 10B with Professor Stankova
Quiz 12; Tuesday, 4/17/2018
Section #211; Time: 11 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. **TRUE** False If two vectors are perpendicular to each other (they form an angle of $90°$), then their dot product is 0.

Solution: The dot product is $\vec{v} \circ \vec{w} = |\vec{v}||\vec{w}| \cos(\alpha)$ but $\alpha = 90°$ and $\cos \alpha = 0$ so the dot product is 0.

2. **TRUE** False If we have found two different solutions to $A\vec{x} = \vec{b}$, then $\det(A) = 0$.

Solution: If we have found two different solutions, then we know that there must be infinitely many solutions so $\det(A) = 0$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) Let $A = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

   (a) (2 points) Calculate $B\vec{v}$.

Solution:
$$B\vec{v} = \begin{pmatrix} -5 \\ -2 \end{pmatrix}$$

   (b) (4 points) Find a solution to $B \begin{pmatrix} x \\ y \end{pmatrix} = \vec{v}$.

Solution: To solve $B\vec{x} = \vec{v}$, we multiply by B^{-1} to get
$$\vec{x} = B^{-1}\vec{v} = \frac{1}{7 \cdot 1 - 2 \cdot 3} \begin{pmatrix} 1 & -2 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 10 \end{pmatrix}$$

(c) (1 point) Is it unique? Why?

Solution: It is unique because $\det(B) \neq 0$.

(d) (3 points) Calculate $\det(A)$.

Solution: We can calculate it as $1 \cdot 2 \cdot 0 + 3 \cdot 1 \cdot (-1) + 4 \cdot 0 \cdot 1 - 1 \cdot 1 \cdot 1 - 3 \cdot 0 \cdot 0 - 4 \cdot 2 \cdot (-1) = 4$.